Single Mechanics – Vector Motion 1

Patrons are reminded that for bodies travelling at *constant velocity*, $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$, where $\mathbf{r}_0$ is the position of the body at the 'start' (i.e. when one's clock is started).

1. At noon Sachin starts at position $(3\mathbf{i} + 4\mathbf{j})$. He travels at a constant velocity of $(-\mathbf{i} + 2\mathbf{j})$ ms^{-1}. Harry starts at a position of $(-10\mathbf{i} + 3\mathbf{j})$. He travels at a constant velocity of $(\mathbf{i} + \mathbf{j})$ ms^{-1}.

   (a) What is Sachin's speed? □

   (b) On what bearing is Harry travelling? □

   (c) Find an expression for Sachin's position after t seconds.

   (d) Do Sachin and Harry meet? If they do meet, give the time and position that they meet. □

2. At noon Toby starts at position $(-\mathbf{i} + 3\mathbf{j})$. He travels at a constant velocity of $(2\mathbf{i} - 4\mathbf{j})$ ms^{-1}. Candice starts at a position of $(21\mathbf{i} - 11\mathbf{j})$. She travels at a constant velocity of $(-\mathbf{i} - 3\mathbf{j})$ ms^{-1}.

   (a) What is Toby's speed? □

   (b) On what bearing is Candice travelling? □

   (c) Find an expression for Toby's position after t seconds.

   (d) Do Toby and Candice meet? If they do meet, give the time and position that they meet. □

   (e) A church lies at position $(-30\mathbf{i} + 3\mathbf{j})$. At what time is Candice due south of the church? □

3. At noon Tom starts at position $(7\mathbf{i} - 89\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{i} + 5\mathbf{j})$ ms^{-1}. Theo starts at a position of $(-11\mathbf{i} + 19\mathbf{j})$. He travels at a constant velocity of $(4\mathbf{i} - \mathbf{j})$ ms^{-1}.

   (a) What is Tom's speed? □

   (b) On what bearing is Theo travelling? □

   (c) Find an expression for Theo's position after t seconds.

   (d) Do Tom and Theo meet? If they do meet, give the time and position that they meet. □

   (e) A school lies at position $(50\mathbf{i} - \mathbf{j})$. At what time is Tom due north of the school? □

4. At noon Pietro starts at position $(26\mathbf{i} + \mathbf{j})$. He travels at a constant velocity of $(-2\mathbf{i} + 7\mathbf{j})$ ms^{-1}. Adriana starts at a position of $(11\mathbf{i} + 9\mathbf{j})$. She travels at a constant velocity of $(\mathbf{i} + 5\mathbf{j})$ ms^{-1}.

   (a) What is Adriana's speed? □

   (b) On what bearing is Pietro travelling? □

   (c) Find an expression for Adriana's position after t seconds.

   (d) Do Pietro and Adriana meet? If they do meet, give the time and position that they meet. □

   (e) A convent lies at position $(-\mathbf{i} + 70\mathbf{j})$. At what time is Adriana due east of the church? □

5. At noon Ed starts at position $(6\mathbf{i} - 6\mathbf{j})$. He travels at a constant velocity of $(\mathbf{i} + \mathbf{j})$ ms^{-1}. Robin starts at a position of $(9\mathbf{i} - 8\mathbf{j})$. He travels at a constant velocity of $(3\mathbf{j} - 2\mathbf{i})$ ms^{-1}.

   (a) What is Ed's speed? □

   (b) On what bearing is Robin travelling? □

   (c) Find an expression for Ed's position after t seconds.

(d) Do Ed and Robin meet? If they do meet, give the time and position that they meet.

□

(e) When $t = 7$, what is the distance between Ed and Robin? □

6. At noon Candy starts at position $(3\mathbf{i} + a\mathbf{j})$. She travels at a constant velocity of $(\mathbf{i} - \mathbf{j})$ ms^{-1}. Kristina starts at a position of $(2\mathbf{i} - 15\mathbf{j})$. She travels at a constant velocity of $(3\mathbf{i} + \mathbf{j})$ ms^{-1}.

(a) What is Candy's speed? □

(b) On what bearing is Kristina travelling? □

(c) Find an expression for Kristina's position after t seconds.

(d) Given that Candy and Kristina do meet, find the value of a. □

(e) When $t = 10$, what is the distance between Candy and Kristina? □

7. (Harder) At noon Cara starts at position $(14\mathbf{i} + 2\mathbf{j})$. She travels at a constant velocity of $(-\mathbf{i} + 6\mathbf{j})$ ms^{-1}. Tori starts at a position of $(-2\mathbf{i} - 5\mathbf{j})$. She travels at a constant velocity of $(3\mathbf{i} + 5\mathbf{j})$ ms^{-1}.

(a) At what time are Cara and Tori closest to each other? $\boxed{t = \frac{57}{17}}$

(b) At this time what is the distance between them? $\boxed{d = \frac{44\sqrt{17}}{17}}$

8. (Harder) At noon Cara starts at position $(-\mathbf{i} - 13\mathbf{j})$. She travels at a constant velocity of $(4\mathbf{i} + \mathbf{j})$ ms^{-1}. Tori starts at a position of $(-2\mathbf{i} + 9\mathbf{j})$. She travels at a constant velocity of $(3\mathbf{i} - \mathbf{j})$ ms^{-1}.

(a) At what time are Cara and Tori closest to each other? □

(b) At this time what is the distance between them? □

9. (Harder) At noon Cara starts at position $(-14\mathbf{i} + 7\mathbf{j})$. She travels at a constant velocity of $(\mathbf{i} - 4\mathbf{j})$ ms^{-1}. Tori starts at a position of $(5\mathbf{i} + 10\mathbf{j})$. She travels at a constant velocity of $(-2\mathbf{j})$ ms^{-1}.

(a) At what time are Cara and Tori closest to each other? □

(b) At this time what is the distance between them? □

10. (Harder) At noon Charity starts at position $(a\mathbf{i} + b\mathbf{j})$. She travels at a constant velocity of $(x\mathbf{i} + y\mathbf{j})$ ms^{-1}. Jenna starts at a position of $(c\mathbf{i} + d\mathbf{j})$. She travels at a constant velocity of $(\theta\mathbf{i} + \phi\mathbf{j})$ ms^{-1}. Find an expression for the time at which they are closest. $\boxed{t = \frac{(x-\theta)(c-a)+(y-\phi)(d-b)}{(x-\theta)^2+(y-\phi)^2}}$

11. (Harder) At noon Ceara starts at position $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$. She travels at a constant velocity of $(-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ ms^{-1}. Lynn starts at a position of $(-2\mathbf{i} - 4\mathbf{j} + 10\mathbf{k})$. She travels at a constant velocity of $(-2\mathbf{i} - \mathbf{j} - 4\mathbf{k})$ ms^{-1}.

(a) At what time are Ceara and Lynn closest to each other? □

(b) At this time what is the distance between them? □